

Expanding $(n + 1)$ -Dimensional Wormhole Solutions in Brans-Dicke Cosmology

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We have obtained two classes of $(n + 1)$ -dimensional wormhole solutions using a traceless energy-momentum tensor in Brans-Dicke theory of gravity. The first class contains wormhole solutions in an open geometry while the second contains wormhole solutions in both open and closed universes. In addition to wormhole geometries, naked singularities and maximally symmetric spacetime also appear among the solutions as special cases. We have also considered the traversability of the wormhole solutions and have shown that they are indeed traversable. Finally, we have discussed the energy-momentum tensor which supports this geometry and have checked for the energy conditions. We have found that wormhole solutions in the first class of solutions violate weak energy condition (WEC). In the second class, the wormhole geometries in a closed universe do violate WEC, but in an open universe with suitable choice of constants the supporting matter energy-momentum tensor can satisfy WEC. However, even in this case the full effective energy-momentum tensor including the scalar field and the matter energy-momentum tensor still violates the WEC.

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I. INTRODUCTION

The scalar-tensor theory of gravity was proposed by P. Jordan for the first time [1] in 1950. Since the Einstein's theory of gravity does not admit Mach's principle, Brans and Dicke tried to modify Einstein's theory in order to incorporate this principle. They presented their theory in 1961 which is now named Brans-Dicke (BD) theory [2]. BD theory describes gravitation through a metric tensor $g_{\mu\nu}$ and a massless scalar field ϕ . In this theory which involves a dimensionless parameter ω , the gravitational constant is a function of space-time coordinates.

Wormholes are objects which connect two distant parts of the same space-time or even two distinct space-times. Although a wormhole solution first entered the physics literature in 1916 [3], the concept was first considered seriously in 1935 by Einstein and Rosen [4] which was later

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called Einstein-Rosen bridge. The word "wormhole" was first time coined by Wheeler [5] in 1957. The main objection against wormholes is that the energy-momentum tensor which supports these geometries turns out to be un-natural. This kind of matter is called 'exotic'. Exotic matter violates the weak energy condition (WEC) and also sometimes other common energy conditions [8, 9]. After the work by Morris and Thorne [6], and subsequent works on traversable Lorentzian wormholes [7] there has been a continued interest in the wormhole issue. Wormholes have been extensively investigated from different points of view in the literature [10, 11, 12, 13, 14, 15, 16, 17, 18].

Recently, it has been discovered that the universe is accelerating [19]. Following this discovery, there has arisen more attention to scalar-tensor theories, including BD as a prototype. The reason is that the scalar degree of freedom in these theories can be used to explain some features of dark energy (the cause of the acceleration). In this paper, we are interested in investigating some exact wormhole solutions of BD theory in a cosmological background. Many authors have considered the wormhole solutions of BD theory. Lorentzian wormholes in Brans-Dicke theory have been analyzed by Agnese and La Camera [20] and Nandi et. al. [21]. Exact rotating wormhole solutions of the BD theory are rarely found and one can see the work by Matos and Nunez [22]. Two classes of massive Lorentzian traversable wormhole solutions in BD theory were found by He and Kim [23]. Some authors have considered observational aspects of wormholes. For example, Cramer et. al [24] investigated the possibility of detecting wormholes by use of their gravitational lensing. Recently Harko et. al.[25] claimed that it is possible to detect wormholes via their electromagnetic radiation spectrum.

Following the inflation theory by A. Guth [26] it has been supposed that the quantum fluctuations in the inflaton field can be assumed as the seed of large scale structures in the universe. Then non-trivial topological objects such as microscopic wormholes may have been formed during that era and then enlarged to macroscopic objects with expansion of the universe. This constitutes part of our motivation for studying wormhole solutions in an evolving cosmological background. To investigate this problem, we use a modified Robertson-Walker (RW) metric and consider the solutions generated by a traceless source.

We organize this paper in the following manner: In Sec. II, we present the ansatz metric and the resulting solutions in $(n+1)$ -dimensions. In section III, we consider different features of the solutions. In section IV, we investigate the corresponding energy-momentum tensor and determine the exoticity parameter. The last section is devoted to conclusions and closing remarks.

II. ACTION, FIELD EQUATIONS AND (N+1)-DIMENSIONAL SOLUTIONS

The action of the Brans-Dicke theory with one scalar field Φ can be written as

$$I_G = -\frac{1}{16\pi} \int_{\mathcal{M}} d^{n+1}x \sqrt{-g} \left(\Phi R - \frac{\omega}{\Phi} (\nabla \Phi)^2 + \mathcal{L}(m) \right), \quad (1)$$

where R is the scalar curvature, \mathcal{L} is the Lagrangian density of the matter and ω is the BD constant. Varying this action with respect to $g_{\mu\nu}$ and ϕ , we obtain

$$\begin{aligned} \phi G_{\mu\nu} &= -8\pi T_{\mu\nu}^M - \frac{\omega}{\phi} \left(\phi_{;\mu} \phi_{;\nu} - \frac{1}{2} g_{\mu\nu} \phi_{;\lambda} \phi^{;\lambda} \right) - \phi_{;\mu;\nu} + g_{\mu\nu} \square \phi, \\ \square \phi &= \frac{8\pi}{(n-1)\omega + n} T_{\lambda}^{M\lambda}, \end{aligned} \quad (2)$$

where $T_{\mu\nu}^M$ is the matter energy-momentum tensor. We are looking for BD wormhole solutions in a cosmological background. To this end, we use the metric

$$ds^2 = -dt^2 + R(t)^2 \left[(1 + a(r)) dr^2 + r^2 d\Omega_{n-1}^2 \right], \quad (3)$$

in which $R(t)$ is the scale factor and $a(r)$ is an unknown function. One will see that $a(r)$ is related to the shape function of the wormhole solutions. With this generalization, our metric is not necessarily homogeneous but it is still isotropic about the center of the symmetry. It is obvious that this metric is an extension of the Robertson-Walker (RW) metric and is less symmetric than it.

In order to start our study, we choose a traceless energy-momentum tensor. With this assumption and equation (2) we have

$$\begin{aligned} G_{\mu}^{\mu} &= \frac{\omega(n-1)}{2\phi^2} \phi_{;\mu} \phi^{;\mu}, \\ \square \phi &= 0. \end{aligned} \quad (4)$$

Here, our task is to solve these equations simultaneously. To this end, we use the ansatz $\phi = \phi(r, t) = S(t)P(r)$. It can be shown that [27], in a cosmological background, $P(r) = \text{constant}$ and we are led to $\phi = \phi(t) = S(t)$. In BD theory $\frac{1}{\phi}$ plays the role of effective gravitational coupling G . Here, we will try a power law scalar field:

$$\phi(t) = At^d, \quad (5)$$

where $A = \text{constant} > 0$, because G , the Newtonian gravitational constant is positive. Considering this ansatz, it turns out that $G_{eff} = \frac{1}{\phi}$, the effective gravitational constant is just as a function of time.

With these assumptions, we are able to solve the equations (4) and find two different classes of solutions.

Class I solutions:

The first class of solutions reads

$$\phi(t) = At^{\frac{-1+\sqrt{n^2-n(n-1)\omega}}{n\omega-(n+1)}}, \quad (6)$$

$$R(t) = C_1 t^{\frac{-1+\sqrt{n^2-n(n-1)\omega}}{n(n\omega-(n+1))} + \frac{1}{n}}, \quad (7)$$

$$1 + a(r) = \frac{1}{1 + \frac{C_2}{r^{n-2}}}. \quad (8)$$

In these solutions, C_1 and C_2 are integration constants. These solutions reduce to those presented in [27] for $n = 3$. It can be easily shown that such a geometry is supported by the following energy-momentum tensor:

$$\rho = -T_t^{Mt} = \alpha_1 At^{\gamma_1}, \quad (9)$$

$$P_r = T_r^{Mr} = \alpha_2 At^{\gamma_1} - \beta_1 \frac{C_2}{C_1^2} \frac{At^{\gamma_2}}{r^n}, \quad (10)$$

$$P_t = T_\theta^{M\theta} = T_\phi^{M\phi} = \dots = \alpha_2 At^{\gamma_1} + \beta_2 \frac{C_2}{C_1^2} \frac{At^{\gamma_2}}{r^n}, \quad (11)$$

where

$$\alpha_1 = \frac{(n((n-2)\omega - (n-1)) + (n\omega - (n-1))\sqrt{n^2 - n(n-1)\omega})}{32\pi^2 G(n\omega - (n+1))^2} = n\alpha_2, \quad (12)$$

$$\beta_1 = \frac{(n-1)(n-2)}{128\pi^2 G} = (n-1)\beta_2, \quad (13)$$

$$\gamma_1 = \frac{(2n+1) + \sqrt{n^2 - n(n-1)\omega} - 2n\omega}{n\omega - (n+1)}, \quad (14)$$

and

$$\gamma_2 = \frac{(n+2)\sqrt{n^2 - n(n-1)\omega} + n(1-2\omega)}{n(n\omega - (n+1))}. \quad (15)$$

Class II of solutions:

The second class of solutions reads

$$\phi(t) = At^{1-n}, \quad (16)$$

$$R(t) = C_1 t, \quad (17)$$

and

$$1 + a(r) = \frac{1}{1 + \frac{C_2}{r^{n-2}} + C_1^2 \left(1 - \frac{(n-1)\omega}{n}\right) r^2}, \quad (18)$$

where, once again, C_1 and C_2 are integration constants related to boundary conditions of our problem and determine what kind of solutions we have. This geometry is supported by the following energy momentum tensor

$$\begin{aligned} \rho &= -T_t^{Mt} = -\eta_1 At^{-(n+1)}, \\ P_r &= T_r^{Mr} = -\eta_2 At^{-(n+1)} - \kappa_1 \frac{C_2}{C_1^2} \frac{At^{-(n+1)}}{r^n}, \end{aligned} \quad (19)$$

$$P_t = T_\theta^{M\theta} = T_\phi^{M\phi} = \dots = -\eta_2 A t^{-(n+1)} + \kappa_2 \frac{C_2}{C_1^2} \frac{A t^{-(n+1)}}{r^n},$$

where

$$\eta_1 = \frac{(n-1)(n+(n-1)\omega)}{64\pi^2 G} = n\eta_2, \quad (20)$$

and

$$\kappa_1 = \frac{(n-1)(n-2)}{128\pi^2 G} = (n-1)\kappa_2. \quad (21)$$

III. PROPERTIES OF THE SOLUTIONS

In order to investigate whether a given solution represents a wormhole geometry, it is convenient to take a look at the metric which is used by Morris and Thorne [6]. In that paper, the metric of the wormhole is written in the form:

$$ds^2 = - dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 [d\theta^2 + \sin^2(\theta) d\phi^2] \quad (22)$$

Where $b(r)$ is the shape function and the throat radius satisfies $b(r_0) = r_0$. If the equation $d(r) = r - b(r)$ has any root r_0 and simultaneously $d(r) > 0$ for $r > r_0$ then we will have a wormhole and r_0 gives the throat radius of the wormhole. Such a geometry can be taken as a wormhole, connecting two distinct universes. We apply such a procedure to our solutions. Since the ansatz metric expands with time, the wormhole throat circumference

$$l = R(t)r_0 \oint d\phi = 2\pi r_0 R(t) \quad (23)$$

expands with time in proportion to the scale factor.

Let us investigate the two classes of solutions separately.

A. Class (I)

According to the equation (8) we have

$$d(r) = 0 \Rightarrow r^{n-2} + C_2 = 0, \quad (24)$$

where C_2 is a constant. It is obvious that this equation has at least one root. In order to study the curvature of this solution, we calculate Kretschman scalar. It turns out that this scalar blows up at the origin ($r = 0$). One can see that for $C_2 < 0$, $d(r)$ has always a root but the radial coordinate r never achieves ($r = 0$) and the geometry in this case represents two open universes, connected by a wormhole. For $C_2 \geq 0$, $d(r)$ has not any root and the r coordinate can reach the origin. In this case, therefore, we have a naked singularity in an open universe.

An interesting point in this class of solutions is that the scale factor, $R(t)$, and the BD scalar field, $\phi(t)$, depend on the BD constant ω while $1 + a(r)$ is independent of ω . This shows that changing the ω , does not affect the geometry of this class of solutions and only changes the expansion rate of the universe and the dynamic of the BD scalar field. It is interesting to note that the $\omega \rightarrow \infty$ limit of this solution yields $\phi(t) = A$ as we expected, while $R(t) \propto t^{\frac{1}{n}}$ which is not identical with the corresponding GR solution.

Let us have a look at the Ricci scalar. The Ricci scalar in $(n+1)$ -dimensions for class (I) solutions reads

$$\mathcal{R} = \frac{\omega(n^2 + 1 - n(n-1)\omega - 2\sqrt{n^2 - n(n-1)\omega})}{(n\omega - (n+1))^2 t^2}. \quad (25)$$

It can be seen that the Ricci scalar doesn't depend on r and is only a function of time.

B. Class (II)

In this subsection, we study its second class of solutions and discuss the geometrical properties. Once again, the Kretschman scalar indicates an intrinsic singularity at $r = 0$.

From equation (18) we have

$$d(r) = 0 \Rightarrow C_1^2 \left(1 - \frac{n-1}{n}\omega\right) r^n + r^{n-2} + C_2 = 0, \quad (26)$$

where C_1 and C_2 are in integration constants. Since we could not solve this equation for general n

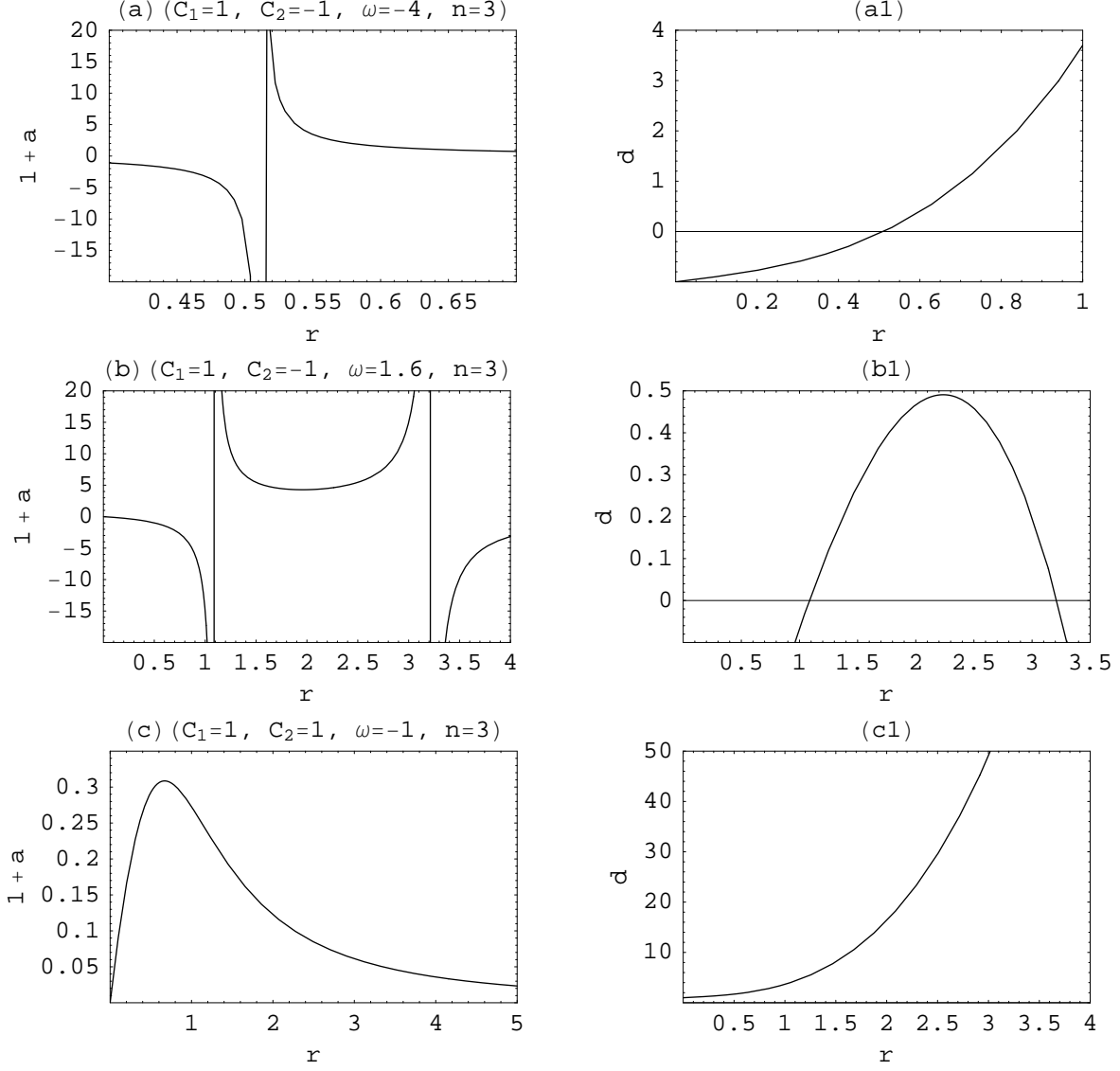


FIG. 1: The behavior of $1+a(r)$ and $d(r)$ for different choices of constants.

analytically, we plot $d(r)$ against r to show the different geometries this solution could represent. We discuss a few specific solutions:

(a) For $C_1 = 1$, $C_2 = -1$, $\omega = -4$, we see from Fig.1 that we have a lower limit on r which corresponds to the throat radius of the wormhole and we see also that there is no upper limit on r which reminds us that we have an open spacetime.

(b) For $C_1 = 1$, $C_2 = -1$, $\omega = 1.6$, we have lower and upper limits on r . The lower limit corresponds to the throat radius of the wormhole and the upper limit signifies a closed spacetime. In this case, we have a wormhole in a closed universe.

(c) $C_1 = 1$, $C_2 = 1$, $\omega = -1$, represents a naked singularity in an open cosmological background,

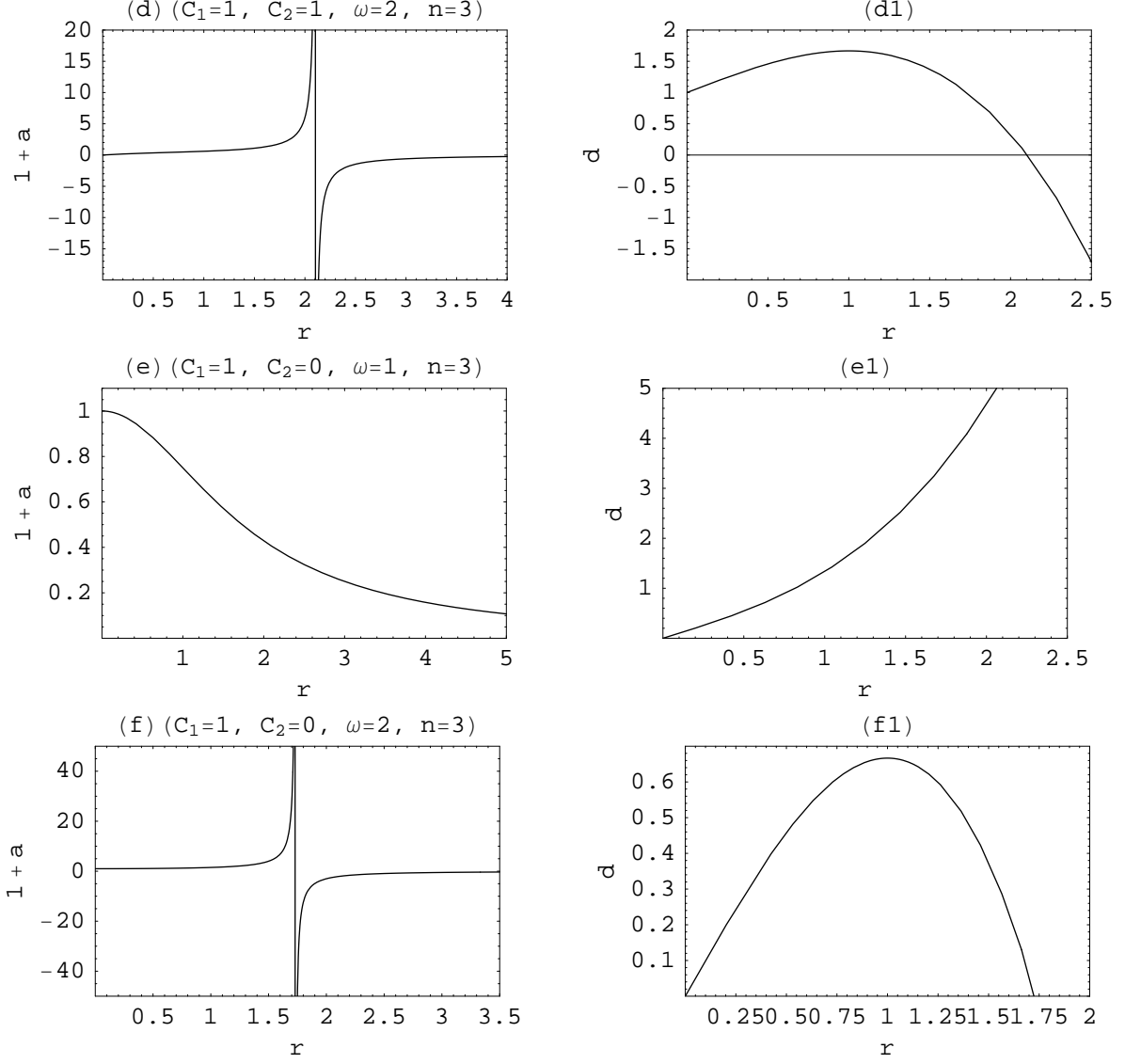


FIG. 2: The behavior of $1 + a(r)$ and $d(r)$ for different choices of constants.

because the Kretschman scalar blows up at the origin ($r = 0$) and we see that the coordinate r can reach the origin.

(d) For $C_1 = 1$, $C_2 = 1$, $\omega = 2$, the solution represents a naked singularity again but this time in a closed universe.

(e) $C_1 = 1$, $C_2 = 0$, $\omega = 1$, leads to a maximally symmetric, open spacetime which corresponds to the *FRW* metric. For other values of ω , too, we have the same behavior.

(f) Finally, the choice $C_1 = 1$, $C_2 = 0$, $\omega = 2$, corresponds to a closed, *FRW* universe.

It is worth seeking the possibility of wormhole solutions with $\omega > 500$ which is motivated by solar system observations. One can see that with suitable choices of C_1 , C_2 and $\omega > 500$ it is

possible to have a wormhole geometry in a closed spacetime, although with an exotic matter.

An interesting point to note is that the value of the ω parameter affects the geometry of the spacetime and changing ω from values smaller than $(\frac{n}{n-1})$ to larger values will change the open geometry to the closed one, but this change doesn't affect the expansion rate $R(t)$ or scalar field $\phi(t)$. From (17), one can see that the wormhole solutions live in a spacetime which expands as $R(t) \propto t$. This scale factor corresponds to the border line between accelerating and decelerating universes. We can also look at the limit ($\omega \rightarrow \infty$) of this class of our solutions and see that in this case, the limit does not approach the GR solutions.

It is worth looking at the Ricci scalar for this class of solutions. It turns out that the Ricci scalar is given by

$$\mathcal{R} = \frac{(n-1)^2 \omega}{t^2}. \quad (27)$$

It can be seen that the Ricci scalar is spatially constant and decreases with time.

C. Wormhole's Two-Way Traversibility

Perhaps the most interesting property of a wormhole is its traversibility. Let us consider this problem in a short discussion. The wormholes presented in this paper are traversable. We present two reasons here. The first reasoning is based on the redshift of a signal emitted at the comoving coordinate r_1 and received by a distant observer. Using the metric (3) and for a radial beam, we obtain

$$\frac{dt}{R(t)} = [1 + a(r)] dr. \quad (28)$$

Using this relation for two signals separated by τ_0 in time when emitted (and τ when detected), we obtain

$$\frac{\tau}{\tau_0} = 1 + z = \frac{R(t_0)}{R(t_1)}, \quad (29)$$

in which $R(t_0)$ is the scale factor at the time of observation, and $R(t_1)$ is the scale factor at the

time of emission. This leads to exactly the same relation as the cosmological redshift relation which shows that the wormhole does not introduce extra (local) redshift. Light signals, therefore can travel to the both sides of the throat and there is no horizon.

The second argument is based on the geodesic equation, which -for the metric (3)- leads to

$$\frac{d^2 r}{d\lambda^2} + \frac{1}{2} \frac{a'(r)}{1+a(r)} \left(\frac{dr}{d\lambda} \right)^2 + 2 \frac{\dot{R}}{R} \frac{dt}{d\lambda} \frac{dr}{d\lambda} = 0 \quad (30)$$

and

$$\frac{d^2 t}{d\lambda^2} + R\dot{R}(1+a(r)) \left(\frac{dr}{d\lambda} \right)^2 = 0, \quad (31)$$

in which λ is an affine parameter along the geodesic. The first equation has the following first integral

$$\frac{dr}{d\lambda} = \frac{C}{R^2 \sqrt{1+a(r)}}. \quad (32)$$

Since the proper distance element is $\delta l = R\sqrt{1+a}\delta r$, we see that there is no radial turning point and any particle can move in either radial directions at any point near to the wormhole, which clearly shows that the wormhole is traversable.

IV. ENERGY-MOMENTUM TENSOR AND THE WEAK ENERGY CONDITION

Let us consider the energy-momentum tensor for different classes of our solutions. Some points are interesting to mention about the energy-momentum tensors which needed for the two cases. Since we are looking for spherical structures in a cosmological background, P_t , P_r and ρ should become almost r -independent at large r . One can easily see that our different solutions have this asymptotic behavior.

As we mentioned in the introduction, the main objection against the plausibility of a wormhole solution is that the energy-momentum tensor which supports this geometry violates the weak energy condition (*WEC*). Here we are interested in investigating *WEC* for the presented classes of solutions.

The weak energy condition requires

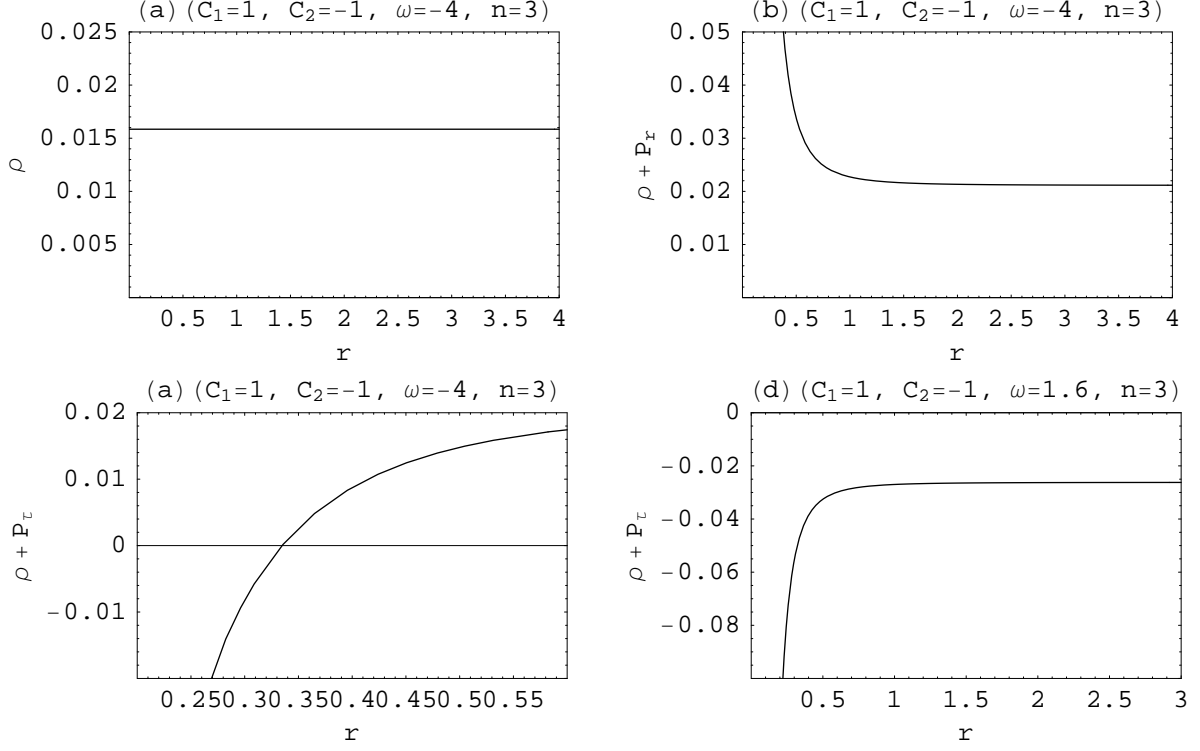


FIG. 3: The energy condition factors are plotted against r . The first three plots are for wormhole in an open universe. The last figure (d) corresponds to $\rho + P_t$ for a wormhole in a closed universe.

$$T_{\mu\nu}u^\mu u^\nu \geq 0 \quad (33)$$

for every nonspacelike u^μ which leads to [28]

$$\rho \geq 0, \quad \rho + P_r \geq 0, \quad \rho + P_t \geq 0. \quad (34)$$

In order to investigate the WEC for class (I), we don't need to consider all the above relations. One can easily see that for class (I), according to (9), the first equation ($\rho \geq 0$) is not satisfied. Then the wormhole solution presented in this case definitely violates the WEC.

Using (34) and the relations (19-20) for the second class of solutions we have

$$\rho \geq 0 \Rightarrow -\frac{(n-1)(n+(n-1)\omega)}{64\pi^2 G} A t^{-(n+1)} \geq 0, \quad (35)$$

$$\rho + P_r \geq 0 \Rightarrow -\frac{(n-1)(2C_1^2(n+1)(n+(n-1)\omega) + n(n-2)C_2r^{-n})}{128\pi^2 G n C_1^2} A t^{-(n+1)} \geq 0, \quad (36)$$

and

$$\rho + P_t \geq 0 \Rightarrow \frac{(-2C_1^2(n-1)(n+1)(n+(n-1)\omega) + n(n-2)C_2r^{-n})}{128\pi^2 G n C_1^2} A t^{-(n+1)} \geq 0. \quad (37)$$

One can see from Fig.(3) that the weak energy condition can be satisfied in the case of wormhole in an open universe which corresponds to $\omega < 0$, and for suitable values of ω . Here, one should note that the supporting matter energy-momentum tensor satisfies the WEC but the effective energy-momentum tensor which includes the BD scalar field and the matter energy-momentum tensor does violate the WEC. We can see from Fig.(3) part (d) that $\rho + P_t$ is everywhere negative and the wormhole in a closed universe always violates the WEC. It is also notable to mention that the wormhole solutions with $\omega > 500$ (as favored by observation) do violate the WEC.

V. SUMMARY AND CONCLUSION

In this paper, we studied exact $(n+1)$ -dimensional, spherical geometries in a cosmological background, in the framework of the BD theory. To this goal, we used a metric which is a simple extension of RW metric and considered a source with traceless energy-momentum tensor. Using these ansatzen, two classes of solutions were found. Class (I), represented wormhole geometries in an open universe. For this class of solutions, the scale factor, BD scalar field and the energy-momentum tensor needed to support the geometry were calculated. It turned out that, changing ω affected the expansion rate $R(t)$ and the scalar field $\phi(t)$. We also calculated the Ricci scalar and saw that it was spatially constant and decreased with time. The second class of solutions was richer than the first one. We classified the solutions in different categories with distinct geometries. Two classes of solutions were shown to represent Lorentzian wormholes in open and closed universes. Two spacetimes containing naked singularity and two maximally symmetric spacetimes were found. For the second class of solutions, we found that the scale factor was proportional to time which corresponded to the border line between decelerating and accelerating universes. In this class, -despite the previous class- we saw that changing ω affected the global geometry of spacetime and did not change the expansion rate $R(t)$ and scalar field $\phi(t)$. Once again, for the second class, we showed that the Ricci scalar was spatially constant and decreased with time. For

both classes of solutions we saw that ρ was only time dependent while P_r and P_t depended on both r and t . In general, one expects that since we are dealing with an inhomogeneous cosmology, pressure and density should depend on both space and time coordinates. Although this is the case for the pressure, it was seen that the density depended only on t as in the homogeneous cosmological models. This unexpected result which came from the field equations, seems to be a property of the BD field equations and does not occur in the Einstein gravity. In addition to that, if we define an average pressure according to ($\bar{P} = \frac{P_r + (n-1)P_t}{n}$), we see that \bar{P} depends only on time, too. We also investigated the traversability of the wormhole solutions and showed that the presented wormholes were traversable. Finally, we considered the energy-momentum tensor for different classes of solutions and investigated the WEC for wormhole solutions. The solutions led to energy-momentum tensors which became almost r -independent as we got far from the central object. We saw that the wormhole solutions presented in class (I) violated the WEC. For the second class, we found out that the matter energy-momentum tensor supporting the wormhole solutions in an open universe with suitable choice of constants could completely satisfy the WEC everywhere. However, the effective energy-momentum tensor was seen to violate the WEC even in this case. Wormhole living in a closed universe always violated WEC.

Acknowledgments

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